Macroeconomic Theory II
Part I: Dynamic macro-equilibrium theory
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Homework #2 (Due January 25, 2005)

1. An economy consists of a (representative) consumer that maximizes utility over consumption and labor and behaves competitively in the rental market and the labor market. His total time endowment in each period is normalized to be one. More specifically, the exogenous stochastic process is a Markovian process \( \{s_t\} \) with transition probability \( \pi(s' | s) \) and initial distribution \( \pi_0 \). Preferences are represented by

\[
E \sum_{t=0}^{\infty} \beta^t u(c, n)
\]

where

\[
u(c, n) = \frac{(c^{1-\alpha} n^\alpha)^{1-\sigma} - 1}{1-\sigma}
\]

The intertemporal budget constraint is

\[
c_t + (1 + \tau^t_t)[k_{t+1} - (1 - \delta)k_t] = (1 - \tau^t_t)\omega_t n_t + r_t k_t + tr_t
\]

where \( tr_t \) are the lump sum transfers to the consumer, \( (1 - \tau^t_t) \) is the labor wedge, and \( 1/(1 + \tau^t_t) \) is the investment wedge. The consumer owns an initial capital stock of \( k_0 \).

The (representative) firm maximizes

\[
A_t(s_t) F(k_t, \gamma_t^t n_t) - r_t k_t - \omega_t n_t
\]

where \( \gamma^t \) is the labor-augmenting technical progress, \( F(k, z) = k^\theta z^{1-\theta} \), and \( A_t(s_t) \) is the efficiency wedge.

The total resource constraint is

\[
c_t + g_t + [k_{t+1} - (1 - \delta)k_t] = A_t(s_t) F(k_t, \gamma_t^t n_t)
\]

where \( g_t \) is the level of (useless) government expenditures.

(a) Discuss briefly what is the meaning of \( \beta, \alpha, \sigma \) and \( \theta \) and give a range of ‘reasonable’ values, according the RBC literature.

(b) Suppose that the government must balance its budget period by period and finances government expenditures and transfers using taxes on labor and investment. Use the above notation to write the government budget.
(c) Show how the competitive decisions of consumers and producers are distorted by the three different wedges. Briefly discuss how wedges are related to the underlying parameters (discussed in a).

(d) Suppose that consumption taxes are introduced (i.e, a consumption wedge \(1 - \tau'_t\)). Briefly discuss how this wedge differs—or is equivalent to—the previous ones.

(e) Now suppose that \(A_t(s_t) = s_t A_{t-1}(s_{t-1})\), with \(A_{t-1}\) given, \(g_t(s_t) = g(s_t)\) and \(\tau'_{t}(s_t) = \tau'\), \(\tau^a_{t}(s_t) = \tau^a\) and \(tr_t\) is whatever needs to be done to balance the government budget. Assume \(\gamma = 1\). Characterize the recursive competitive equilibrium of this economy.

(f) Briefly discuss why this benchmark economy can be useful to study actual historical economies.

2. Use your "two OCDE economies" of the Homework 1 to complete this exercise [NOTE: If needed, this exercise can be presented on February 1st]

(a) In homework 1 you were asked to report some basic ‘stylized facts’ regarding GDP (per capita & constant prices), etc. As you may have seen, many RBC articles report ‘stylized facts’ regarding GNP. Does it make much difference whether you use GDP or GNP data, both empirically (use your data) and theoretically? If so, which measure should we use in studying RBC?

(b) (A premier on ‘costs of business cycles’). Using the data of one of your OCDE countries you can have a first measure of how costly business cycles may be. To do so, take the above specification of preferences with some suitable parameters (taking \(\sigma = 1\) and \(\alpha = 0\) will simplify matters) and then you can compute the realized present value utility along the realized path of consumption and labor. However, having concave utility the agent will rather have smooth consumption (and hours worked, but we can abstract from this to start). Therefore you can compute the average consumption in the sample period—say, \(\bar{c}\)—and the corresponding utility that would have been achieved with this consumption. You can then compute the constant consumption—say, \(\bar{c}'\)—that will make the consumer indifferent between the realized consumption and having such constant consumption through the sample period. Finally, you compute a measure of RBC cost for your sample economy: \(\frac{c - c'}{\bar{c}}\).

(c) Now use your ‘favorite specification’ of technology parameters to do some TFP (business cycle) accounting (you can even do that with Excel). In particular, given the data on capital (you should have constructed this one from investment data), hours worked and output (recall a!), you can now compute the efficiency wedge as residual. Briefly report any possible interesting fact regarding such efficiency wedge. Is it very sensitive to your choice of parameters?
(d) [Optional] You can further refine your answer to (b). In particular, you can compute RBC costs taking into account a ‘more realistic’ choice of parameters, although this may involve making some consideration regarding the behavior of $n$. Similarly, you may take as benchmark not constant consumption, but ‘smooth consumption’ as given by your –possibly, HP– filter. That is, only consider the costs from removing cycles with respect to a –possibly, non linear– trend.

(e) [Optional] You can further elaborate in your answer to (c). For your OCDE countries gather data on ‘effective taxes’ (one tax may be enough) and compute the resulting wedge. Show how taking into account of such wedge improves your description of the data using an RBC model.

3. RMT Exercise 8.8 (not in 1st ed.) [Optional: Exercise 8.9 d]

4. [Optional] RMT Exercises 8.10 and 8.11 (not in 1st ed., although eq. (8.5.4) is (7.8), (8.7.1) is (7.17), and chapter 13 is chapter 10, in the 1st ed.) [NOTE: If needed, this exercise can be presented on February 1st]