1. [Follow up from exercises 1 & 2 of Homework #2] An economy consists of a (representative) consumer that maximizes utility over consumption and labor and behaves competitively in the rental market and the labor market. His total time endowment in each period is normalized to be one. There is no uncertainty and preferences are given by

\[ u(c, 1 - n) = \frac{[c^{1-\alpha}(1 - n)^\alpha]^{1-\sigma} - 1}{1 - \sigma} \]

The intertemporal budget constraint is

\[ (1 + \tau^c_t)c_t + b_{t+1} + [k_{t+1} - (1 - \delta)k_t] \leq (1 - \tau^n_t)\omega_t n_t + r_t k + b_t(1 + i_t) \]

where \( \tau^c_t \) is the labor tax rate at \( t \), \( \tau^n_t \) the consumption tax rate, \( i_t \) the nominal interest rate on one-period government bonds, \( b_t \). The consumer owns an initial capital stock of \( k_0 \) an an initial level of public debt of \( b_0 = 0 \).

The (representative) firm maximizes

\[ A_t F(k_t, \gamma^n_t n_t) - r_t k_t - \omega_t n_t \]

where \( \gamma^n_t \) is the labor-augmenting technical progress, \( F(k, z) = k^\theta z^{1-\theta} \), and \( A_t \) is the efficiency wedge.

The total resource constraint is

\[ c_t + g_t + [k_{t+1} - (1 - \delta)k_t] = A_t(s_t)F(k_t, \gamma^n_t n_t) \]

where \( g_t \) is the level of (useless) government expenditures.

(a) Define the government intertemporal budget constraint and show that both the consumer and the government intertemporal budget constraints can be transformed into present value budget constraints—with Arrow-Debreu prices.
(b) Characterize the competitive equilibrium of this economy

(c) Solve for the Ramsey plan

(d) Suppose that \( \gamma = 1 \) and \( g_t = g > 0 \) and suppose that the Ramsey plan has reached a steady state. Characterize the ‘optimal tax policies’. Show that, in fact, there is a menu of consumption and labor ‘optimal tax policies’ [Optional: show that, in fact, the Ramsey plan converges to a steady state]

(e) Is there a ‘time consistency problem’?

(f) Now consider one of your ‘favorite OCDE economies’ and, using some of the work previously done, give reasonable parameters to this economy. However, now you must provide a first approximation to what the government budget can be (here there is no money!), as to have an idea of what the corresponding tax rates can be. How does the corresponding Ramsey plan compares with the realized allocation? [Note: this is a ‘first approximation’ or ‘back of the envelope’ calculation, but you should try your best with the available data. Again, if needed the answer to this question can be postponed one week to February 8].

2. RMT Exercises 15.7 (not in 1st ed.).

(g) Finally, do not assume that tax rates must be zero in period zero, but may be bounded. Briefly discuss the ‘time consistency problem.’