1 Dynamic Setup

Based on the static framework developed in the first part of the paper, we now draw on Diamond [3] and develop a stylized OLG model to analyze contract dynamics and to characterize the way in which shocks are amplified or dampened through the financial sector. More specifically, we embed the contracting problem analyzed in the first part of the paper into the simplest possible OLG framework, with zero population growth, inelastic supply of savings and full depreciation of the capital stock.

At any point in time, a new generation of measure one is born: of this generation, a measure λ^G is assumed to be composed of $G-type$ entrepreneurs, a measure λ^B is composed of $B-type$ entrepreneurs and the remaining measure is composed of households. Thus, $\lambda^G + \lambda^B < 1$. Both households and entrepreneurs maximize consumption in their old age, where the latter are risk neutral and the preferences of the former are inessential in exactly the same way as was assumed for the static analysis. All members of each generation supply inelastically one unit of labor when they are young, which is combined with capital from the old to produce a consumption good according to a constant returns to scale technology denoted by

$$y_t = \theta g(1, k_{t-1}) \tag{1}$$

where g is a continuously differentiable and concave constant returns to scale function, k is per-capita stock of capital (and aggregate stock, due to the normalization on population size) and its subscript denotes the date of birth of the generation that owns it. Both old and young are paid a competitive price for supplying their factors of production, so that the young receive,

$$w_t(k_{t-1}) = \theta[g(k_{t-1}) - k_{t-1}g'(k_{t-1})] \tag{2}$$

for their labor while the old receive

$$q_t(k_{t-1}) = \theta g'(k_{t-1}) \tag{3}$$

per unit of capital, where q_t denotes the marginal productivity of capital in the production of the final good at time t.

Since everyone is interested in consuming only during old age, the income of the young is saved in its entirety, while that of the old is fully consumed. Thus, at any point in time, total savings of this economy will be equal to w_t. This amount is deposited in banks, which serve as intermediaries and channel it to entrepreneurs by means of the separating contracts derived in the previous section.

Entrepreneurs then borrow in order to produce capital according to their respective technologies, and their payments to banks are ultimately distributed among depositors.[1]

With this basic setup in mind, we will now formalize the discussion by defining an intertemporal equilibrium. For expositional convenience, we express loans and investment in terms of the final good whereas repayment and collateral are expressed in terms of capital. Abusing notation, we use

$$\rho_t^e = \frac{r_t}{q_{t+1}^e}$$

to denote the ratio between the interest rate at time t and the expected price of capital at time $t+1$. The latter expectation is formed according to perfect foresight.

Definition 1. *For a given initial value w_0, an intertemporal equilibrium of the asymmetric information economy is defined as a trajectory*

$$\{k_t, w_t, q_{t+1}^e, r_t, C^{EQ}(w_t, \rho_t^e) : t \geq 0\}$$

such that, for all t

1. $C^{EQ}(w_t, \rho_t^e) = C^{SEP}(w_t, \rho_t^e)$ *as characterized in Proposition* **??**
2. *There is perfect foresight in forming expectations regarding the price of capital*
3. *Labor and capital market clearing conditions (2) and (3) are satisfied*
4. $\{I_t^B, I_t^G\} \in C^{SEP}(w_t, \rho_t^e)$ *satisfy*

$$\lambda^G I_t^G(w_t, \rho_t^e) + \lambda^B I_t^B(\rho_t^e) = w_t \tag{4}$$

In the case of full information, the definition of equilibrium is analogous to the previous one, the only difference being that the equilibrium contracts equalize marginal productivities across technologies.

Since ρ_t^e in this economy essentially depends on the total amount of savings and on the way in which it is allocated across technologies, we can rewrite the equilibrium conditions of this economy

[1]Note the similarity with [1] and [2] in that the source of wealth for entrepreneurs is the wage they receive while young.

at time t as the following system of equations

$$k_t = K(\beta_t, w_t) \tag{5}$$

$$w_t = \varpi(k_{t-1}, \theta) \tag{6}$$

$$F(\beta_t, w_t) = 0 \tag{7}$$

where β_t denotes the share of total savings allocated to the G technology. Hence, (5), (6) and (7) represent, respectively, the production of capital given investment, the equilibrium wage given the capital stock and the relationship between savings and its allocation under the separating regime as implied by equilibrium contracts.

By considering the relationships implied by (6) and (7), we can specify (5) as

$$k_t = K(\beta_t(w_t(k_{t-1})), w_t(k_{t-1})) \tag{8}$$

where β_t is defined as before and K denotes the gross return of investment as expressed by,

$$\lambda^G \alpha^G p^G f(\frac{\beta_t(w_t(k_{t-1})) \cdot w_t(k_{t-1})}{\lambda^G}) + \lambda^B \alpha^B p^B f(\frac{(1-\beta_t(w_t(k_{t-1}))) \cdot w_t(k_{t-1})}{\lambda^B}) \tag{9}$$

Equation (9) shows how the amount of capital produced depends on the total amount of savings - $w_t(k_{t-1})$ - both directly and indirectly. The latter effect arises from the impact of wages on equilibrium contracts and, consequently, on the way in which credit is allocated among different technologies. The accumulation path of capital in the economy is then given by,

$$\frac{dk_t}{dk_{t-1}} = (\frac{\partial K}{\partial \beta_t}\frac{d\beta_t}{dw_t} + \frac{\partial K}{\partial w_t})\frac{\partial w_t}{\partial k_{t-1}} \tag{10}$$

With this basic setup in mind, we analyze the dynamics of the model in the presence of full information. After that, we will turn our attention the dynamics of the asymmetric information economy when equilibrium contracts are separating.

1.1 Full Information Dynamics

The present subsection briefly analyzes some aspects of the (trivial) dynamics of the model under full information. In the latter setting, loan sizes will satisfy

$$\alpha^G p^G f'(I_t^G) = \alpha^B p^B f'(I_t^B)$$

in all periods. Thus, not surprisingly,

$$\frac{\partial K}{\partial \beta_t} = \alpha^G p^G f'(I_t^G) w_t(k_{t-1}) - \alpha^B p^B f'(I_t^B) w_t(k_{t-1}) = 0 \quad (11)$$

meaning that, since the marginal productivity of investment is equalized across different technologies, changes in the allocation of deposits have no marginal effects on the gross return to investment. By replacing (11) into (10), we can express the rate of capital accumulation as

$$\frac{dk_t}{dk_{t-1}} = [\beta_t \alpha^G p^G f'(I_t^G) + (1-\beta_t)\alpha^B p^B f'(I_t^B)] w_t' = \alpha^G p^G f'(I_t^G) w_t' > 0 \quad (12)$$

so that the current capital stock affects the future stock by the increase in wages adjusted by the marginal productivity of investment. In such a scenario, a steady state is given by a level of capital k^* such that,

$$k^* = \lambda^G \alpha^G p^G f(\frac{\beta(w(k^*) \cdot w(k^*)}{\lambda^G}) + \lambda^B \alpha^B p^B f(\frac{(1-\beta(w(k^*)) \cdot w(k^*)}{\lambda^B}) \quad (13)$$

Stability of such a steady state in turn requires,

$$\frac{dk_t}{dk_{t-1}} = \alpha^G p^G f'(I_t^G) w_t' < 1 \quad (14)$$

Since we are interested in analyzing the dynamics of the asymmetric information scenario, we assume that the baseline full information economy has a unique, stable and nonoscillatory steady sate.

1.2 Asymmetric Information Dynamics

We now analyze the case of asymmetric information. We focus on the case with binding collateralization constraints, which is evidently the one of economic interest.

The main difference that arises with the full information economy is that now, the allocation of deposits among different technologies (β) will be constrained by the presence of collateral. In our setting, in which the only source of wealth for young entrepreneurs are wages, this means that β will be a function of the latter not only as determinants of deposits but also in their role as entrepreneurial wealth. Thus, capital accumulation in the asymmetric information economy will be given by,

$$\frac{dk_t}{dk_{t-1}} = [\beta_t \alpha^G p^G f'(I_t^G) + (1-\beta_t)\alpha^B p^B f'(I_t^B)]w_t' \tag{15}$$
$$+[\alpha^G p^G f'(I_t^G) - \alpha^B p^B f'(I_t^B)]w_t' \frac{d\beta_t}{dw_t} w_t$$

where the first and second terms on the right hand side represent, respectively, the productivity of savings generated by additional capital and the change in allocative efficiency stemming from the variation in wealth. The first term, which reflects the weighted marginal productivity of investment, will by definition be lower in the presence of asymmetric information and binding wealth constraints (since in such a scenario the allocation of funds will be inefficient). The second term, on the other hand, describes the change in the proportion of credit allocated to the $G - type$ sector: not surprisingly, this term may in principle be positive or negative depending on conditions which will be thoroughly analyzed below. An obvious consideration is that, if contracts eventually achieve the efficient allocation, the growth rate of the asymmetric information economy will be equal to that of the benchmark economy. In such a situation, the marginal productivity of investment will be equalized across sectors and growth will thus depend only on the latter.

For any given level of capital, then, the productivity of investment will be lower in the presence of asymmetric information due to the inefficiency induced by the equilibrium contracts.[2] Moreover, the proportion of funds allocated to the B technology may itself decrease or increase with economic growth, depending on the way in which the latter affects the incentive compatibility constraint. Let us note that the asymmetric information economy has at least one stable steady state:

Lemma 1. *If the full information economy has a unique, stable steady state with a capital stock of k^*, the asymmetric information economy under the separating regime will display at least one stable steady state with $\hat{k} \leq k^*$.*

Proof. Regardless of the presence of asymmetric information, we know that $f'(k) \to \infty$ as $k \to 0$. On the other hand, it is also the case that $f'(k) \to 0$ as $k \to \infty$. Finally, for a given initial condition, the capital accumulation path under the separating regime is bounded from above by its full information counterpart. Therefore, this economy must display at least one stable steady state which cannot have a higher capital stock than k^*. □

[2] In what remains of this application, we refer somewhat loosely to the "efficiency" of contracts in terms of β, the proportion of funds allocated to the S technolgy.

1.2.1 Evolution of Investment under the Separating Contracts

Thus far we have analyzed the evolution of capital as a function of w and β, without specifying the relationship between these two variables. We now analyze this relationship in order to characterize contract dynamics, paying particular attention at the case in which $I^G < I^B$: as we saw in the static framework, this case corresponds to an economy in which $\frac{w}{I^B} < 1$.

We differentiate the equilibrium mapping (7) and obtain the following relationship between total savings and the share invested in the G technology,

$$\frac{d\beta_t}{dw_t} = \frac{[(1-\frac{p^B}{p^G}) - (\frac{\beta_t}{\lambda^G})[\alpha^B p^B f'(\frac{\beta_t w_t}{\lambda^G}) - \frac{p^B}{p^G}\rho_t^e] - p^B\alpha^B f''(I_t^B)(\frac{(1-\beta_t)}{\lambda^B})[I_t^B - \frac{p^B}{p^G}I_t^G - w_t[1-\frac{p^B}{p^G}]]}{(\frac{w_t}{\lambda^G})[\alpha^B p^B f'(\frac{\beta_t w_t}{\lambda^G}) - \frac{p^B}{p^G}\rho_t^e] - p^B\alpha^B f''(I_t^B)(\frac{w_t}{\lambda^B})[I_t^B - \frac{p^B}{p^G}I_t^G - w_t[1-\frac{p^B}{p^G}]]} \tag{16}$$

Remark 1. *The terms in (16) have the following signs:*

1. $[\alpha^B p^B f'(\frac{\beta_t w_t}{\lambda^G}) - \frac{p^B}{p^G}\rho_t^e] > 0$ *always, regardless of relative loan sizes;*
2. $[(1-\frac{p^B}{p^G}) - (\frac{\beta_t}{\lambda^G})[\alpha^B p^B f'(\frac{\beta_t w_t}{\lambda^G}) - \frac{p^B}{p^G}\rho_t^e]$, *is the slope of the NMC with the sign changed. Thus, when $I^G < I^B$ this term is negative, and it is positive when the opposite inequality holds;*
3. $-p^B\alpha^B f''(I_t^B)[I_t^B - \frac{p^B}{p^G}I_t^G - w_t(1-\frac{p^B}{p^G})]$, *which is found both in the numerator and the denominator, will be negative or positive depending on the relative size of loans. When $I^B > I^G$, the term will be positive, whereas it will become negative if the inequality is reversed.*

The first two expressions in the previous remark are the first-order effects on contracts, while the third one is a second-order effect induced by changes in the interest rate. If we consider an economy in which $I^B > I^G$ and analyze (16) in light of the remark, we can say the following:

a) The denominator is positive. An increase in β_t has no first order effects on $B - type$ profits while it raises the profits associated to $G - type$ contracts, therefore exerting pressure on the incentive compatibility constraint. Additionally, there is a second order effect associated to the increase in interest rates that accompanies a reallocation of funds towards the G-sector. When $I^B > I^G$, this effect reinforces the previous one by increasing the relative cost of $B - type$ contracts.

b) Regarding the numerator, the first order effects (given by the first two terms) are negative. This stems, once again, from the fact that changes in the overall level of credit (while keeping β_t constant) have first-order implications only for the profitability of $G - type$ contracts:

in particular, this effect makes the latter more attractive whenever $I^B > I^G$. Additionally, increases in the volume of credit have a second order effect through the changes in interest rate that they induce: this effect tends to make $B-type$ loans relatively more (less) attractive whenever $I^B > I^G (<)$.

(a) and (b) together imply that, in the presence of sufficiently small second-order effects (i.e., if the marginal productivity of investment does not fall too rapidly), an economy with loan sizes $I^G < I^B$ will allocate an increasing proportion of its resources to the $B - type$ technology as it expands. This implies that such an economy would never revert to the "right" loan sizes as it grows, thereby displaying a steady state with less capital than its full information counterpart and in which the $B - type$ technology invests more than $G - type$ entrepreneurs.

1.3 Financial Dampening

From our previous analysis, we identified two effects of an increase in wages. On one hand, higher wages increase the amount of savings in the economy and - consequently - the amount of investment and the marginal productivity of labor. This effect, which reinforces the original increase in wages, is at work both in the full information benchmark and in our adverse selection economy. On the other hand, there is a second effect which is consequential exclusively in the latter: an increase in wages affects the allocation of savings between technologies. If entrepreneurial wealth is low relative to investment, that this effect may work against the original increase in wages by expanding the proportion of funds allocated to the less productive technology.

We analyze these effects more closely by evaluating the long-run effects of a productivity shock in the full and asymmetric information economies. To do so, we work with the wage mapping (6), which we rewrite to account for (5) and (7) to deliver

$$w_t = \theta[g(k_{t-1}(\beta_t(w_{t-1}), w_{t-1})) - k_{t-1}(\beta_t(w_{t-1}), w_{t-1})g'(k_{t-1}(\beta_t(w_{t-1}), w_{t-1}))]$$

By Lemma 1, we know that the wage mapping has at least one stable steady state under asymmetric information. If we denote $\hat{w}$ to be such a steady-state in our economy, we can evaluate the long-run effect of a productivity shock in the presence of asymmetric information by considering the following elasticity,

$$\hat{\xi}_{\hat{w},\theta} = \frac{\partial \hat{w}/\partial \theta}{\hat{w}/\theta} = \frac{1}{1 + \hat{k}\theta g''(\hat{k})\frac{d\hat{k}}{d\hat{w}}} \quad (17)$$

When the economy faces a productivity shock, the steady state level of wages is affected directly through the increase in labor productivity but also indirectly through the increase in the stock of capital. This increase in the capital stock is itself affected by the increase in wages, which expands savings and hence capital accumulation. These effects are captured by (17) where the impact of wages on the capital stock is given by

$$\begin{aligned}\frac{d\hat{k}}{d\hat{w}} &= [\hat{\beta}\alpha^G p^G f'(\hat{I}^G) + (1-\hat{\beta})\alpha^B p^B f'(\hat{I}^B)] + \\ &\quad [\alpha^G p^G f'(\hat{I}^G) - \alpha^B p^B f'(\hat{I}^B)]\frac{d\hat{\beta}}{d\hat{w}}\hat{w}\end{aligned} \tag{18}$$

A first effect, positive and captured by the first line of (18), represents the increase in investment generated by the greater availability of funds. A second effect represents the reallocation of funds that takes place as a consequence of contract dynamics. This aspect, which is captured by the second line in (18), is positive or negative depending on the way in which $\hat{\beta}$ is affected by wages.

In the case of the full information economy, for which we denote steady state levels by the superscript *, the analogous elasticity is,

$$\xi^*_{w^*,\theta} = \frac{\partial w^*/\partial\theta}{w^*/\theta} = \frac{1}{1+\theta k^* g''(k^*)\frac{dk^*}{dw^*}} \tag{19}$$

Proposition 1. $\hat{\xi}_{\hat{w},\theta} < \xi^*_{w^*,\theta}$ *iff*

$$\hat{k}\cdot\left|g''(\hat{k})\right|\frac{d\hat{k}}{d\hat{w}} < k^*\cdot\left|g''(k^*)\right|\frac{dk^*}{dw^*} \tag{20}$$

When comparing the elasticities $\xi^*_{w^*,\theta}$ and $\hat{\xi}_{\hat{w},\theta}$, there are different effects which act in opposite direction under standard assumptions. Since under asymmetric information the steady state entails a relatively low level of capital and output, the productivity of capital and - for a given allocation of credit - investment is higher, tending to amplify the effects of shocks. On the other hand, though, the relatively inefficient allocation of investment across technologies, plus the fact that this inefficiency may even increase as a consequence of the shock, tend to dampen the effects of the latter under asymmetric information. If this effect is sufficiently strong, the adverse selection economy will dampen the impact of exogenous shocks, and this will be the case even though entrepreneurs' wealth will be positively correlated with the latter.

In terms of (20), under the standard assumption by which $k\cdot|g''(k)|$ is decreasing in k, the increase in wages induced by an increase in the capital stock is greater in the adverse selection economy. In such a scenario, then, a necessary condition for (20) to be satisfied is that the increase in the capital stock directly induced by an expansion in savings is higher under full than under asymmetric information as captured in (18). In other words, the inefficiency in the allocation of investment and the evolution of this inefficiency as dictated by contract dynamics in (16) must be substantial enough to offset the concavity effects.

Ultimately, it is the endogeneity of the interest rate that potentially dampens shocks in the presence of adverse selection. It is essentially this feature, which forces contracts to clear the market while being incentive compatible, that may lead to a decrease in the efficiency of investment as a consequence of growth. In an economy in which the interest rate is fixed at r, there is no direct impact of wages on investment under full information. In the adverse selection economy under the separating regime, though, an expansion in wages increases the amount that G entrepreneurs can invest and, consequently, leads directly to an increase in the stock of capital. In such a scenario, exogenous perturbations will necessarily be amplified by the presence of asymmetric information.

To summarize, in the full information economy a positive shock increases the amount of period t savings, raising investment while maintaining marginal productivities equalized across sectors. With adverse selection, however, there is an additional effect that must be taken into account. First of all, investment is allocated inefficiently across technologies. Moreover, as (15) illustrates, an increase in wages will affect not just the total amount of investment but also its allocation between sectors, since β will also change as credit expands. When an increase in collateralizable wealth cannot be fully absorbed by $G-type$ entrepreneurs without violating incentive compatibility, the interest rate must fall and $B-type$ investment must expand in order to restore market clearing. It is thereby the feature inherent to adverse selection, by which the notion of incentive compatibility restricts joint as opposed to individual allocation, what generates the possibility of financial dampening in the presence of an exogenous shock.

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