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By ANDREU MAS-COLELL *and* ASSAF RAZIN

I. Introduction

OUR main purpose in this paper is to show how some of the patterns of growth of a dual economy studied by Fei and Ranis [4], Jorgenson [6, 7], and Dixit [2] can be explained by a simple neoclassical growth model. Such patterns of growth include: a decreasing rate of migration from rural to urban sector; a stage of accelerated accumulation of capital; etc. In order to accomplish this objective it is necessary to introduce explicitly migration into the framework of neoclassical growth models. Therefore we shall assume that labour cannot be transferred instantaneously between sectors. Furthermore, the rate of migration will be determined by economic forces. In this respect our model is different from the one studied by Harris and Todaro [5] in which the industrial wage is institutionally fixed. There is an unavoidable trade-off between simplicity and generality. Since we regard the simplicity of the model as one of its most compelling aspects we shall not attempt to generalize and we shall use freely special assumptions. For example, following Jorgenson [6] we shall assume that production functions are of Cobb–Douglas form.[1]

The plan of this paper is as follows. In Section II we shall introduce a model of migration and capital accumulation for an economy with agricultural and industrial sectors. We shall show that the direction of migration between sectors is completely determined by the proportion of the total labour force occupied in agriculture. In Section III we shall analyse rates of growth of migration, capital accumulation, industrial output–capital ratio, and the terms of trade. In Section IV we shall discuss a policy of subsidy-tax. The steady states generated by various tax rates will be derived and those which are 'inefficient' will be singled out. We shall also analyse the effect of policy on the length of the period of migration.

II. The model

Consider an economy with two productive sectors, an agricultural sector A producing output for consumption and an industrial sector I producing output for consumption and investment.[2] The production functions are

assumed linear homogeneous of Cobb–Douglas form. The total labour force is a constant fraction of total population (assumed for convenience to be one). Thus, *per capita* outputs, denoted by y, can be written:[1]

$$y_I = \rho k_I^\beta, \qquad y_A = (1-\rho)k_A^\alpha, \tag{1}$$

where k_I, k_A are capital–labour ratios in sector I and A respectively and ρ is the proportion of total labour force employed in the industrial sector.

There exists full employment of capital and labour,

$$\rho k_I + (1-\rho)k_A = k, \tag{2}$$

where k is the capital–labour ratio available for the economy. Let p be the price of the industrial good in terms of agricultural good. Assuming that capital is instantaneously transferred, competition will equalize the marginal productivity of capital in both sectors

$$p\beta k_I^{\beta-1} = \alpha k_A^{\alpha-1} = r. \tag{3}$$

Denoting the wage rate by w, competition within sectors implies

$$w_I = p(1-\beta)k_I^\beta, \qquad w_A = (1-\alpha)k_A^\alpha. \tag{4}$$

Per capita national income (in units of agricultural good) is expressed by $y_A + py_I$. Postponing a more extensive discussion for the next paragraph, we shall now assume, for concreteness, a constant ratio of saving to income s, and also that a constant proportion of income δ is being spent on the industrial good for consumption purposes. Therefore, demand for industrial output is $s(y_A + py_I) + \delta(y_A + py_I)$; supply of industrial output is py_I. In equilibrium we have

$$(\delta + s)(y_A + py_I) = py_I. \tag{5}$$

Let $\lambda = s/s + \delta$ be the proportion of total industrial output in the form of new capital goods. The solution of (1)–(3) and (5) is given by

$$k_I = \theta\frac{k}{\rho}, \qquad k_A = (1-\theta)\frac{k}{1-\rho} \tag{6}$$

where $\qquad \theta = \dfrac{\beta s}{\beta s + \alpha(\lambda - s)} = \dfrac{(s+\delta)\beta}{(s+\delta)\beta + (1-s-\delta)\alpha}.$

This shows that the momentary equilibrium is well defined.

The model accommodates a more general situation where the fraction of the aggregate demand for industrial goods in total income is not necessarily constant over time. Assume that the population consists of three groups of consumers: the wage earners in the agricultural sector A, the wage earners in the industrial sector I, and the owners of capital C. Every group allocates a constant proportion of its income (not necessarily the same between groups) to consumption and investment in industrial goods.

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[1] The multiplicative constants of the Cobb–Douglas functions are eliminated by an appropriate choice of the units of labour and agricultural output.

Let ν_i, μ_i stand for the proportion of income of group i spent on industrial goods for consumption and investment purposes, respectively; define $\chi_i = \nu_i + \mu_i, i = A, I, C$. Then, for example, if $\nu_I > \nu_A$, i.e. if the propensity to consume industrial goods is greater for industrial workers as compared with agricultural workers, the proportion of aggregate income spent on industrial goods for consumption purposes will increase when migration takes place.

Therefore, *per capita* demand for industrial output is

$$(1-\rho)\chi_A w_A + \rho\chi_I w_I + \chi_C rk;$$

supply of industrial output is py_I. In equilibrium we have

$$(1-\rho)\chi_A w_A + \rho\chi_I w_I + \chi_C rk = py_I. \tag{7}$$

Substituting (1), (2), (3), and (4) into (7) we obtain the same expression as in (6), where θ is substituted by

$$\theta' = \frac{(1-\alpha)\beta\chi_A + \alpha\beta\chi_C}{(1-\alpha)\beta\chi_A + \alpha(1-\chi_I(1-\beta))}. \tag{8}$$

Similarly, easy computation shows that the proportion of total industrial output in the form of new capital goods is a constant λ' given by

$$\lambda' = \frac{(1-\theta')\beta(1-\alpha)\mu_A + \theta'(1-\beta)\alpha\mu_I + \beta\alpha\mu_C}{\alpha\beta'}. \tag{9}$$

Hence the simple and the more general models are formally similar. Throughout the rest of the paper we shall analyse the simple model.

Population is increasing at a constant relative rate n. Therefore, capital–labour ratio will be accumulated according to

$$\dot{k}/k = \lambda y_I/k - n \tag{10}$$

$$= \lambda\theta\beta\left(\frac{k}{\rho}\right)^{\beta-1} - n$$

where use has been made of the momentary equilibrium conditions in (6).

We assume that migration of labour[1] is positively related to wage differential

$$\dot{\rho}/\rho = f(w_I, w_A), \tag{11}$$

where f is a continuously differentiable function such that

$$\text{sign} f = \text{sign}(w_I - w_A).$$

The motion of the system can be conveniently analysed in the phase diagram of Fig. 1.

The stationary law of migration is solved by setting the right-hand side

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[1] If M is the rate of migration into the industrial sector and L_I is the industrial labour then

$$\frac{M}{L_I} = \frac{\dot{L}_I - nL_I}{L_I} = \frac{\dot{L}_I}{L_I} - n = \frac{\dot{\rho}}{\rho}.$$

of (11) equal to zero, i.e. $w_I = w_A$. Using (4) and (6) we conclude that no migration will take place if:

$$\hat{\rho} = \frac{\alpha(1-\beta)\theta}{\theta(1-\beta)\alpha+(1-\theta)(1-\alpha)\beta}.^1 \tag{12}$$

From (11) we see that $\dot{\rho} \gtrless 0$ as $\rho \lessgtr \hat{\rho}.$ \hfill (13)

We illustrate this finding with a numerical example. Let $s = 0.15$, $\lambda = 0.20$, $\delta = 0.60$, $\alpha = 0.30$, and $\beta = 0.40$, then $\hat{\rho} \approx 0.70$. Therefore as
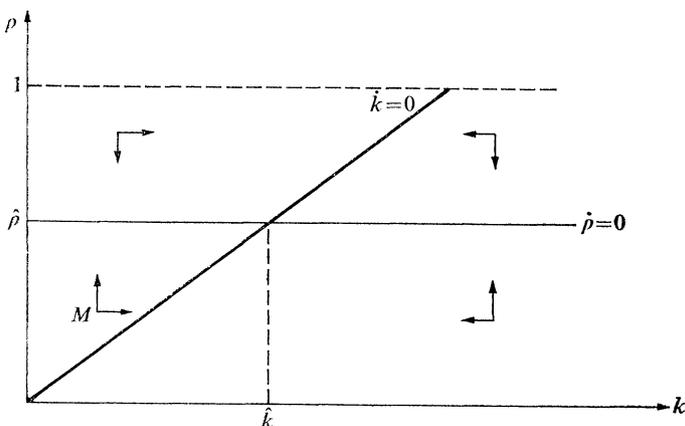

Fig. 1

long as the proportion of population occupied in industry is less than 0.70 migration to the industrial sector will take place.[2]

The differential equation (10) has a stationary solution when

$$k = c\rho, \tag{14}$$

where

$$c = \left(\frac{\lambda}{n}\right)^{1/(1-\beta)} \theta^{\beta/(1-\beta)}.$$

As can be seen from Fig. 1 and (10) and (13) the economy has a unique and globally stable steady state: $\rho = \hat{\rho}$, $\hat{k} = c\hat{\rho}$.

III. Relative rates of growth

In order to pursue the analysis further we need a specific form for the migration equation (11). The following migration equation is similar to the ones used by M. Todaro [9], P. Zarembka [11], and satisfies the plausible requirement that $\dot{\rho}/\rho \to \infty$ as $w_I/w_A \to \infty$. Moreover, this equation behaves nicely in our model.

$$\dot{\rho}/\rho = \gamma\left[\frac{w_I - w_A}{w_A}\right] \tag{15}$$

where γ is a positive constant.

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[1] In general, when production functions are not of Cobb–Douglas form the proportion $\hat{\rho}$ will depend on k. See [1].

[2] In the more general model let $\alpha = 0.3$, $\beta = 0.4$, $\mu_A = \mu_I = 0$, $\nu_A = 0.4$, $\nu_I = 0.75$, $\mu_C = 0.1$, then, again, $\rho \approx 0.7$.

Hereafter we shall restrict our analysis to region M in Fig. 1, where capital is being accumulated and migration of labour into industry takes place. We shall discuss in this section the implications of the model for relative rates of growth of migration, capital accumulation, output–capital ratio in the industrial sector, and the terms of trade.

1. The relative rate of growth of migration $(\dot\rho/\rho)$.[1] Substituting (3)–(4) and (6) into (15) we obtain

$$\dot\rho/\rho = \gamma\left[\frac{(1-\beta)}{\beta}\frac{\alpha}{(1-\alpha)}\frac{(1-\rho)}{\rho}-1\right].$$ (16)

Clearly the rate of growth of migration decreases when migration into the industrial sector takes place (i.e. ρ increases).
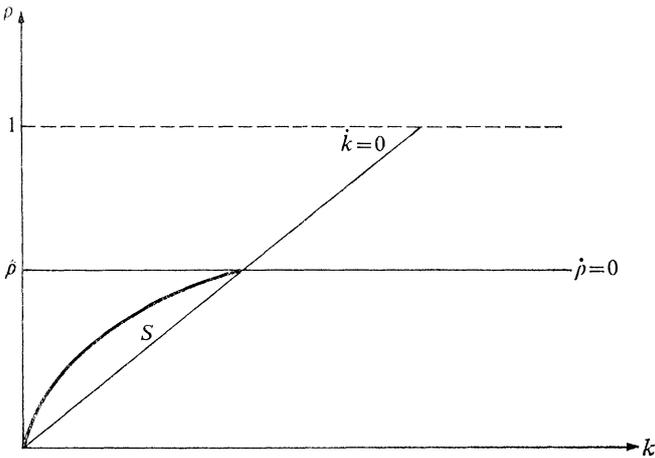

FIG. 2

2. The relative rate of growth of capital accumulation. Denoting this rate by $\dot K/K$ we have $\dot K/K = \dot k/k+n$. Differentiating (10) with respect to time[2] and taking (16) into account we can draw in Fig. 2 the locus

$$\frac{d(\dot k/k)}{dt} = 0.$$

This locus cannot intersect the $\dot k = 0$ or $\dot\rho = 0$ curves. (Furthermore, for any k there exists a ρ such that (k, ρ) is below this curve and for any ρ there exists a k such that (k, ρ) is to the left of it.)

Referring to Fig. 2 any path which starts initially in region S will exhibit

[1] Observe that $\dot L_I/L_I$, the rate of growth of industrial labour force, is equal to $(\dot\rho/\rho)+n$. Recently Sato and Niho [8] have studied a model of a dual economy where population growth is related to the level of *per capita* income. Their expression for rate of migration (equation (17)) therefore includes as an argument also *per capita* income. Their approach, however, poses the problem that changing the numeraire by which *per capita* income is measured will change the pattern of migration they attempt to study.

[2] From (10) we have $\dfrac{d\dot k/k}{dt} = (\dot k/k)\lambda(1-\beta)(\dot\rho/\rho-\dot k/k).$

initially a phase of accelerated capital accumulation but eventually will enter a phase of decreasing rate of growth of capital as the economy approaches the steady state. This is shown in Fig. 3.

3. The industrial output–capital ratio (z). From (1) and (6)

$$z = \theta^{\beta-1}(k/\rho)^{\beta-1}.$$

Comparing this expression with (10) yields the conclusion that $\dot{K}/K$ and z move together over time as it is indicated in Fig. 3.
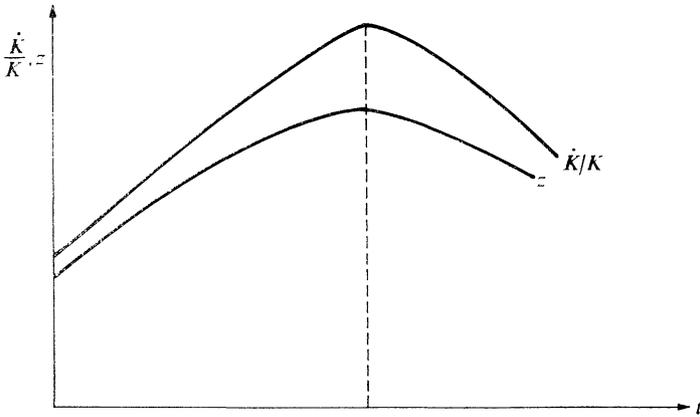

FIG. 3

4. The terms of trade of agriculture ($1/p$). From (3) and (6) we see that the terms of trade of agriculture are proportional to $k^{\beta-\alpha}\rho^{1-\beta}(1-\rho)^{\alpha-1}$. Clearly if the industrial sector is more capital intensive then the terms of trade will move monotonically in favour of agriculture.

In particular our model is capable of generating the patterns of growth of migration and capital accumulation reported by Dixit [2].

IV. Policy implications

1. We shall show briefly how policy variables may be introduced in a simple way into the model. Suppose an *ad valorem* subsidy (tax) at the rate of τ is given to the agricultural sector. Suppose that the government raises (gives) these funds from an income tax.

Equation (3) becomes

$$(1+\tau)\alpha k_A^{\alpha-1} = p\beta k_I^{\beta-1}, \quad \tau > -1. \tag{17}$$

The rest of the model is unchanged.

Let us define
$$\theta_\tau = \frac{s\beta}{s\beta+(\lambda-s)\alpha(1+\tau)}.$$

In equation (6) we substitute $\theta\tau$ for θ.

In Fig. 4 we represent the locus of possible steady states as the subsidy rate τ ranges over $(-1, +\infty)$.[1]

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[1] This is equivalent with $\theta\tau$ ranging over $(0, 1)$.

Point A in Fig. 4 represents a steady state corresponding to a higher subsidy rate than the steady state represented by B. However, some points on this locus will represent steady states which are inefficient for the economy. We say that a steady state is efficient if there are no other steady states with more of consumption (*per capita*) of both goods.[1]

Per capita consumption of industrial goods increases monotonically with ρ along this locus (as can be inferred from (1) and (6)). *Per capita* consumption of the agricultural good is maximized when $\rho = \alpha$ on this locus.[2] Therefore any steady state where $\rho < \alpha$ is inefficient.
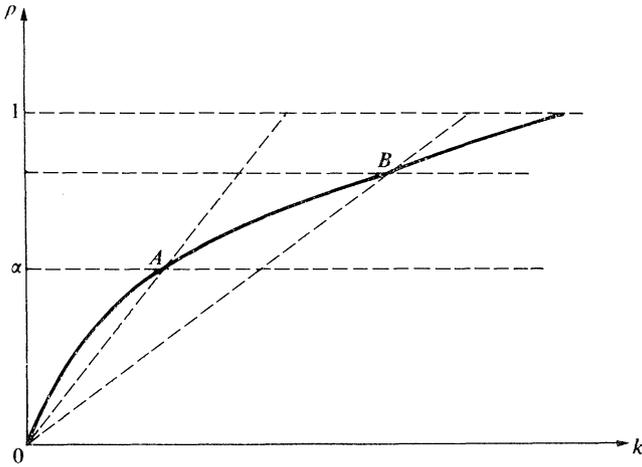

FIG. 4

2. The simplicity of the model enables us to solve explicitly for the time paths of ρ and k. This means that the period of time needed to reach some objectives of ρ and k, which can be controlled by the government through τ, is readily computable. The solution for the differential equation (16)[3] is given by

$$\rho_t = (\rho_o - \hat{\rho})e^{-\gamma(\hat{\rho}/1 - \hat{\rho})t} + \hat{\rho} \tag{18}$$

The explicit time path of k_t can be easily found by using the transformation $q = k^{\beta-1}$.

Suppose an amount of time $\bar{t}$ is needed for the economy to reach a given composition of labour force $\bar{\rho}$. What will be the effect of an increase in the rate of subsidy on the period in which the same labour force composition will be reached? To answer this question we differentiate totally the

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[1] If, as in the model studied by Dixit [3], saving by market forces is assumed to be socially suboptimal then points on the locus of possible steady states other than the efficient points may become targets for the project evaluator.

[2] This is the well-known 'Golden Rule'. Among steady states, $\lambda \rho k_I^\beta = nk$ the consumption of agricultural good is maximized when $\beta \lambda k_I^{\beta-1} = n$. Combining these with (2)–(4) we get $\rho = \alpha$.

[3] Observe that the differential equation (12) does not depend on k.

right-hand side of (18) and set it equal to zero, thereby obtaining $dt/d\tau > 0$. Thus, subsidizing agriculture will lengthen the migration period.

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