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CORRECTIONS TO AN EQUILIBRIUM EXISTENCE THEOREM FOR A GENERAL MODEL WITHOUT ORDERED PREFERENCES

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It has been brought to our attention that the proof of the equilibrium theorem in Gale and Mas-Colell (1975) has two gaps. First: The particular bound used to truncate consumption sets may be too small to guarantee the non-empty valuedness of the modified budget sets defined at the bottom of page 13 (we are indebted to H. Cheng for pointing this out to us). Second: The augmented preference mappings may not, as claimed, have an open graph [we are indebted to J. Foster (1978) for pointing this out to us]. The theorem and the main lines of the proof remain valid. The first problem can be repaired by taking a sufficiently large truncation of the consumption sets; the second, either by noticing that the weaker property of lower-semicontinuity of the augmented preference maps suffices, or by a slight change of the definition of the augmented preference maps. Details follow.

Consumption sets: We claim there is a vector $r > f - e$ such that defining $\hat{X}_i = \{x_i | x_i \in X_i \text{ and } x_i \leqq r\}$ the correspondence $\gamma_i(p)$ is non-empty valued on Δ^* as asserted at the top of page 14 of our paper. Note first that there is a finite set $X_i' \subset X_i$ such that $\min pX_i' < \alpha_i(p)$ for all $p \in \Delta^*$. Indeed, for every $p \in \Delta^*$ let $x_p \in X_i$ be such that $px_p < \alpha_i(p)$. By the continuity of α_i there is, for every $p \in \Delta^*$, an open neighborhood of p, $V_p \subset \Delta^*$, such that $qx_p < \alpha_i(q)$ for every $q \in V_p$. The family of open sets $\{V_p : p \in \Delta^*\}$ covers Δ^* and since Δ^* is compact there is a finite subcover $\{V_{p_1}, \ldots, V_{p_s}\}$. Take then $X_i' = \{X_{p_1}, \ldots, X_{p_n}\}$. If we now pick r sufficiently large so that $X_i' \subset \hat{X}_i$ our claim is established.

Augmented preference maps, first approach: First, a definition. Let Z be a non-empty subset of some euclidean space. A mapping $\xi : Z \to Z^2$ is *lower semicontinuous* if $z' \in \xi(z)$ and $z_n \to z$ implies the existence of $z_n' \in \xi(z_n)$ with $z_n' \to z'$. The fixed point theorem of section 2 of our paper remains true if we replace the expression 'whose graphs are open in $X \times X_i$' by 'which are lower semicontinuous'. The proof needs no modification because the selection theorem of Michael which is used requires only a lower semicontinuous correspondence.

Now, our augmented preference maps $\hat{P}_i$ are trivially seen to be lower semicontinuous. Further, this property is inherited by the mappings ϕ_i defined on page 14. Hence, Lemma 3 holds true and the rest of the proof proceeds as in the paper.

We may remark that with this line of proof hypothesis (9) can be weakened to 'the preference mappings are lower semicontinuous, irreflexive (that is, $x_i \notin P(x_i)$), and non-empty, open, convex valued'.

We always felt a little uneasy about appealing to the powerful Michael theorems to get a selection for an open graph correspondence since for this case one could be constructed quite simply. As it turns out, it was good we allowed some slack!

Augmented preference maps, second approach: Let $\tilde{X}_i$ be the affine space spanned by X_i. Replace our definition of the augmented preference maps by $\hat{P}_i(x)=P_i(x)\cup\{\lambda x+(1-\lambda)y : y\in \mathrm{Int}_{\tilde{X}} P_i(x), 0<\lambda<1\}$. Then $\hat{P}_i$ has all the properties claimed by us and again the equilibrium obtained with $\hat{P}_i$ is an equilibrium for the original P_i. We are indebted to Kurt Hildenbrand for this approach.

References

Foster, J., 1978, A counterexample to the open graph property of the augmented preference mappings as claimed by Gale and Mas-Colell, May (Cornell University, Ithaca, NY).

Gale, D. and A. Mas-Colell, 1975, An equilibrium existence theorem for a general model without ordered preferences, Journal of Mathematical Economics 2, 9–15.