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THE PRICE EQUILIBRIUM EXISTENCE PROBLEM IN TOPOLOGICAL VECTOR LATTICES[1]

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A price equilibrium existence theorem is proved for exchange economies whose consumption sets are the positive orthant of arbitrary topological vector lattices. The motivation comes from economic applications showing the need to bring within the scope of equilibrium theory commodity spaces whose positive orthant has empty interior, a typical situation in infinite dimensional linear spaces.

KEYWORDS: Walrasian equilibrium, price equilibrium, existence, topological vector lattice, Riesz space, proper preferences.

1. INTRODUCTION

IN THIS PAPER we reconsider the price equilibrium existence problem for exchange economies with an infinite number of commodities and finitely many consumers. Our purpose is to do so in a context sufficiently general to encompass as particular instances a number of commodity spaces that have been found useful in applications. For example: (i) $L_\infty(M, \mathcal{M}, \mu)$, or L_∞ for short, the space of essentially bounded measurable functions on a measure space (see Bewley (1972)), which is relevant to the analysis of allocation of resources over time or states of nature; (ii) $ca(K)$, the space of countable additive signed measures on a compact metric space (see Mas-Colell (1975), and Jones (1984a)), which has been exploited for the analysis of commodity differentiation; (iii) $L_2(M, \mathcal{M}, \mu)$, the space of square integrable functions on a measure space (cf. Duffie-Huang (1983a), Chamberlin-Rothschild (1983), Harrison-Kreps (1979)), which arises in finance economics.

For a survey of the finite dimensional existence theory, see Debreu (1982). The classical reference for the infinite dimensional theory is Bewley (1972). Mathematically, a crucial feature of his analysis (and of his generalizations (Bojan (1974), El'Barkuki (1977), Toussaint (1984), Florenzano (1983), Ali Khan (1984), Yannelis-Prabhakar (1983)) is that the consumption set, which is the natural positive orthant of the commodity space, L_∞ in his case, has a nonempty norm-interior. It is on account of this that spaces such as $ca(K)$ or L_2 are not covered by his work and that a more general, unifying approach is called for. Jones (1984a) has offered an existence theorem for $ca(K)$. We mention, as an incidental remark, that the initial stimulus for our research was an attempt to understand the role of the differentiability-like hypotheses that in contrast to Bewley (1972) appear in Jones (1984a): are they required to guarantee the existence of a price functional or to yield one with a particular important property, i.e., continuity

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on characteristics? Another thought-provoking paper was Ostroy (1984) where L_1 spaces are considered.

The mathematical setting for our commodity spaces will be *topological vector lattices.* Those turn out to be sufficiently general for our aims and also tractable. Commodity spaces which are vector lattices, or Riesz spaces, have been introduced by Aliprantis and Brown (1983) in the context of an excess-demand approach to equilibrium. A topological vector lattice is a vector lattice where the lattice operations $(x, y) \mapsto x \vee y$ are (uniformly) continuous. A most important example are the *Banach lattices.* Those are Banach spaces equipped with a positive orthant which induces a lattice order $\geq$ with the property that $|x| \leq |y|$ implies $\|x\| \leq \|y\|$. Here $|x| = (x \vee 0) - (x \wedge 0)$ is the absolute value of x. See Aliprantis and Burkinshaw (1978), and Schaefer (1971). All the examples mentioned above (and many more, e.g., all the l_p and L_p spaces) are Banach lattices when equipped with their natural norms and orders. A Banach space which is not a Banach lattice is $C^1([0, 1])$. A vector lattice which is not a Banach space is R^∞. A major surprise of this paper is precisely this relevance of lattice theoretic properties to the existence of equilibrium problem. One would not have been led to expect it from the finite dimensional theory. In the latter it is possible to formalize and solve the existence problem using only the topological and convexity structures of the space (cf. Debreu (1962)). Informally, a source for the difference seems to be the following: in general vector lattices, order intervals (i.e., sets of the form $\{x\colon a \leq x \leq b\}$) are, as convex sets, much more tractable and well behaved than general bounded, convex sets. An unfortunate consequence of this is that the exchange case, the one we handle, is inherently simpler than the general production case (see Section 9).

As for the technique of proof, we follow the Negishi (1960) approach and carry our fixed point argument on the utility possibility frontier. This is particularly sensible in a model with infinitely many commodities but only a finite number of consumers. For L_∞ this line of attack was proposed and implemented by Bewley in a regretably unpublished paper (1969), and then by Magill (1981). Their papers have been quite helpful to our analysis. Notably, Bewley already asserted that "it can be expected that this method of proof may be applied to infinite dimensional commodity spaces other than L_∞."

The paper, and the proof of the eventual result, is divided in four parts which we now describe briefly. Each part tackles a conceptually distinct issue. It is worth noting that the first three are trivial in the finite dimensional case.

After short sections on commodities and prices, Section 4 takes care of a preliminary technical and conceptual step, namely, it attacks and solves, rather straightforwardly, the one consumer problem. Only separation arguments are involved.

Sections 5 and 6, which involve no essentially new idea, define and study the properties of the utility possibility set of the economy; in particular, its closedness.

Section 7 is the heart of the paper. We give conditions on individual preferences and the commodity space for weak optima to be supportable by nontrivial price systems. It should be emphasized that this does not follow automatically from the well-behavedness of single consumer problems (i.e., from the solution in Section 4). It is in this section, and only in this section, that the lattice properties of the space are exploited.

Finally, Section 8 states a quasiequilibrium existence theorem and carries out the fixed point proof in the utility possibility frontier. The arguments are identical to those of Bewley (1969) and Magill (1981).

Section 9 mentions possible extensions and comments on some of the recent literature.

2. THE COMMODITY SPACE

The commodity space, denoted L, is a (real) vector space endowed with two structures: a linear order, denoted $\geqslant$, and a linear topology. We impose the weak regularity conditions that the topology be Hausdorff and locally convex and that the positive cone of the space $L_+ = \{x \in L\colon x \geqslant 0\}$ be closed. Technically: L is a locally convex, Hausdorff *ordered vector space.*

This formalization is general enough to cover every case of interest. More restricted closes of commodity spaces will be considered in subsequent sections.

3. PRICE FUNCTIONALS

We shall take as price valuation functions the elements of L^*, i.e., the continuous linear functionals on L, This is very general. It is worth noting that economic consideration will often lead us to positive linear functionals and that in some spaces of interest (e.g., Banach lattices, see Section 7) those are automatically continuous (see Schaefer (1974, p. 84), for Banach lattices).

4. THE SINGLE CONSUMER ECONOMY

Suppose we have a single consumer characterized by the *consumption set* $L_+ = \{x \in L\colon x \geqslant 0\}$, the *endowment vector* $\omega \in L_+$ and a *preference relation* $\succsim$ on L_+ which we take to be *reflexive, transitive, complete, continuous* (i.e., the set $\{(x, y) \in L_+ \times L_+\colon x \succsim y\}$ is closed), *convex* (i.e., the set $\{y \in L\colon y \succsim x\}$ is convex for every $x \in L$), and *monotone* (i.e., for every $x \in L_+$, $y \geqslant x$ implies $y \succsim x$ and there is $v \geqslant 0$ such that $x + \alpha v \succ x$ for all $\alpha > 0$).

With the above hypothesis it is well known that in the finite dimensional case, i.e., when $L = R^n$, the vector ω can always be sustained as a *price quasiequilibrium*, i.e., there is $p \neq 0$ such that $z \succsim \omega$ implies $p \cdot z \geqslant p \cdot \omega$. The next example shows that this ceases to be true in the infinite dimensional case.

EXAMPLE 1: Let $K = Z_+ \cup \{\infty\}$ be the compactification of the positive integers. Our commodity space will be $L = ca(K)$, i.e., the space of signed bounded countably additive measures with the bounded variation norm. For $x \in L$ and $i \in K$ let $x_i = x(\{i\})$.

For every $i \in K$ define a function $u_i : [0, \infty) \to [0, \infty)$ by

$$u_i(t) = \begin{cases} 2^i t & \text{for } t \leqslant \frac{1}{2^{2i}}, \\ \frac{1}{2^i} - \frac{1}{2^{2i}} + t & \text{for } t > \frac{1}{2^{2i}}. \end{cases}$$

See Figure 1.

The preference relation $\succsim$ on the consumption set L_+ is then given to us by the concave utility function $U(x) = \sum_{i=1}^{i=\infty} u_i(x_i)$. It is an easy matter to verify that $U(x)$ is well defined, strictly monotone (i.e., $x \geqslant y$, $x \neq y$ implies $x \succ y$) and weak-start continuous (i.e., U is continuous for the weak convergence for measures).

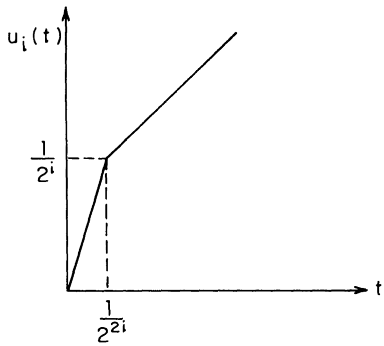

FIGURE 1

Let now $\omega \in L_+$ be defined by $\omega_i = 1/2^{2i+1}$ for $i < \infty$ and $\omega_\infty = 1$. We claim that the only linear functional p such that $p \cdot z \geqslant p \cdot \omega$ whenever $z \succsim \omega$ is the trivial $p = 0$. This is a consequence of the unboundedness of the sequence of marginal utilities $u_i'(\omega_i) = 2^i$, $i < \infty$. Indeed, let p be such a functional. For any $x \geqslant 0$ we have $\omega + x \succsim \omega$. Hence, $p \cdot x \geqslant 0$, i.e., p is a positive linear functional. For $i \in K$ denote $p_i = p \cdot e_i$, where $e_i(\{j\}) = 1$ if $j = i$ and $= 0$ otherwise. Suppose first that $p \cdot \omega > 0$. Then, by the usual marginal rate of substitution arguments, we should have $p_1 > 0$ and $p_i = p_1 z^{i-1}$, $i < \infty$, $p_\infty = \frac{1}{2} p_1$. Define $z \in L_+$ by $z_i = 1/p_i$ and $z^n \in L_+$ by $z_i^n = z_i$, if $i \leqslant n$, and $z_i^n = 0$ otherwise. Then $z - z^n \geqslant 0$ which implies $p \cdot z^n \leqslant p \cdot z$ for all n. But $p \cdot z^n = \sum_{i=1}^n p_i z_i = n > p \cdot z$ for n sufficiently large. Contradiction. The only possibility left is $p \cdot \omega = 0$. Pick an arbitrary $z \geqslant 0$ and let $z^m \in L_+$ be defined by $z_i^m = z_i$ if $m < i < \infty$ and $z_i^m = 0$ otherwise. For every m we have $0 \leqslant \alpha(z - z^m) \leqslant \omega$ for some $\alpha > 0$. Hence $p \cdot (z - z^m) = 0$ for all m. The sequence z^m converges to 0 in the (variation) norm on $ca(K)$. For this linear topology in $ca(K)$ any positive linear functional is continuous (see Section 3). Therefore, $p \cdot z^m \to 0$ and so $0 = \lim_m p \cdot z^m = p \cdot z$. Because $L = L_+ - L_+$ we conclude $p \cdot z = 0$ for any $z \in L$, i.e., $p = 0$.

Obviously, if positive results are to be obtained for N consumer economies, they will have to apply to the one consumer case. So the situation of the previous example has to be ruled out. *We shall do so, essentially, by assumption.* Specifically, and informally, we shall impose a priori bounds on the marginal rates of substitution displayed by admissible preference relations with respect to a given composite bundle of commodities.

The next definition can be looked at as a strengthening of monotonicity.

DEFINITION: The preference relation $\succsim$ is *proper* at $x \in L_+$ if there is $v \geqslant 0$ and an open neighborhood of the origin V with the property that $z \in L$ and

$$x - \alpha v + z \succsim x \quad \text{implies} \quad z \in \varepsilon V.$$

We say that $\succsim$ is *uniformly proper* if $\succsim$ is proper at every $x \in L_+$ and v, V can be chosen independently of x.

As indicated, properness at x merely says that the improvement direction v is desirable in the sense that it is not possible to compensate for a loss of αv with a vector which is too small relative to αv. Observe that the vectors z such that $x - \alpha v + z \notin L_+$ do not create any problem. They pass the test vacuously.

Geometrically, properness at x means that there is an open cone $\Gamma \subset L$ containing a positive vector (hence Γ is nonempty) such that $(-\Gamma) \cap \{z - x \in L_+ : z \succsim x\} = \varnothing$. See Figure 2. Properness at x is automatically satisfied if there is a continuous, positive functional p such that $p \cdot z \geqslant p \cdot x$ whenever $z \succsim x$. Indeed, take $v \geqslant 0$ and V such that $p \cdot v > 0$ and $z \in V$ implies $|p \cdot z| < |p \cdot v|$. Conversely, if $\succsim$ is convex, then properness at x implies the existence of such a functional (see Section 7). Thus, as intended, in the convex case properness at a point is equivalent to the property we want to have. Uniform properness requires that the same cone work at all x.

If L_+ has a nonempty interior (e.g., $L = L_\infty$ with the sup. norm), then uniform properness is automatically implied by monotonicity. It may also happen that, at least at a point, the property is implied by monotonicity, convexity, and continuity; it follows from Bewley (1972) that this is the case for strictly positive points of L_∞ with the Mackey topology. But in general Example 1 has established

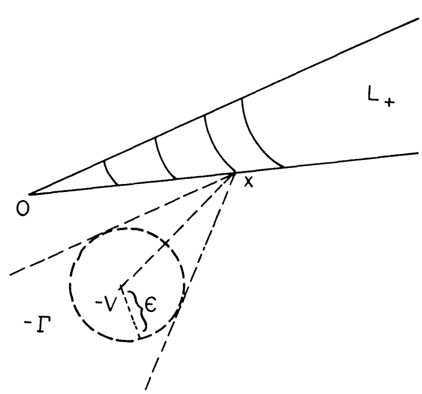

FIGURE 2

that properness at a point is an additional hypothesis. The relation among properness, uniform properness and the existence of supporting prices is expanded in Section 7.

The properness property has precedents. Debreu and Hildenbrand (see Bewley (1972)) suggested the use of a boundedness condition on marginal utilities. In the context of an optimal growth problem in Hilbert space, Chichilnisky-Kalman (1980) imposed a condition analogous to properness at a point. Of course, Jones (1984a) should also be mentioned. Simultaneously and independently of us, a similar tool has been used by Ostroy (1984). It appears that our property is intimately related to the extendability of the presence relation $\gtrsim$ to the entire space as a continuous, convex, monotone preference relation. See Richards (1985a, 1985b) for this. Extendible preferences have been used in Chichilniski-Heal (1984) and Mas-Colell (1985b).

5. A DIGRESSION ON UTILITY THEORY

As a topological space, L_+ need not be second countable. Our two reference spaces L_∞ and $ca(K)$ are not so in general. Thus even if the preference relation $\gtrsim$ on L_+ is continuous, we cannot appeal to Debreu's theorem (1954) in order to represent $\gtrsim$ by a (continuous) utility function. However, the monotonicity of $\gtrsim$ can be exploited for that purpose in the manner of Kannai (1970). Because continuous utility representations are extremely convenient technical tools, we digress to present such a result.

PROPOSITION 1: *Let $\gtrsim$ be a continuous preference relation on a subset X of an ordered vector space L of the form $X=\{x\in L: a\leqslant x\leqslant b\}$ for two fixed $a, b\in L$. Suppose that for any $x, y\in X$, $x\geqslant y$ implies $x\gtrsim y$. Then there is a continuous utility function $u: X\to[0,1]$ such that $x\gtrsim y$ if and only if $u(x)\geqslant u(y)$.*

PROOF: This follows by standard arguments in utility theory. If $X=\varnothing$ or $a\gtrsim b$ there is nothing to prove. So, let $b\geqslant a$ and $b\succ a$. Consider first the set $J=\{\alpha a+(1-\alpha)b: 0\leqslant\alpha\leqslant 1\}$. Because J is topologically a segment there is a continuous $f: J\to[0,1]$ such that $f(0)=0$, $f(1)=1$ and f represents the restriction of $\gtrsim$ to J. For any $x\in X$ let $v(x)$ be such that $v(x)\in J$ and $v(x)\sim x$. Because J is connected and a subset of the union of closed sets $\{z: z\gtrsim x\}$, $\{z: x\gtrsim z\}$, such a $v(x)$ must exist. Let then $u(x)=f(v(x))$. Obviously u is a utility function. To see that it is continuous take any $t\in[0,1]$. Since $u(X)=[0,1]$ there is $x\in X$ such that $u(x)=t$. Then the sets $u^{-1}([t,\infty))=\{z: z\gtrsim x\}$, and $u^{-1}((-\infty,t])=\{z: x\gtrsim z\}$ are closed by the continuity of $\gtrsim$. *Q.E.D.*

The above Proposition is all we shall need. It does not imply, however, that a $\gtrsim$ defined on L_+ admits a utility function on the entire L_+. See Monteiro (1985) and Shafer (1984) for recent investigations on utility theory for continuous, monotone preferences in linear ordered spaces.

6. EFFICIENT ALLOCATIONS AND THE PARETO FRONTIER

Suppose we have N consumers with preferences $\succsim_i$ on the consumption set L_+. There is also a total endowment vector ω. We assume that every $\succsim_i$ is continuous and monotone. To avoid degeneracy, we take ω to be desirable, i.e., $\alpha\omega \succ_i 0$ for all $\alpha > 0$ and $i \in N$.

Let $X = \{z \in L_+ : 0 \leqslant z \leqslant \omega\}$. A vector $x \in X^N$ such that $\sum_{i=1}^N x_i \leqslant \omega$ is called a *feasible allocation.* Denote by $\hat{X} \subset X^N$ the set of feasible allocations.

By Proposition 1 there is, for every $i \in N$, a continuous utility function $u_i : X \to [0, 1]$ with $u_i(0) = 0$, $u_i(\omega) = 1$. Define $U : X^N \to [0, 1]^N$ by $U(x) = (u_1(x_1), \dots, u_N(x_N))$. Denote by Δ the closed $N-1$ simplex and, for any $s \in \Delta$, let $f(s) = \sup\{\alpha \in R : \alpha s \in U(\hat{X})\}$. Then:

PROPOSITION 2: *$f: \Delta \to [0, N]$ is a well-defined continuous function. Moreover, $f(s) > 0$ for all $s \in \Delta$.*

PROOF: $f(s) > 0$ for all $s \in \Delta$ is an obvious consequence of the desirability of ω.

Let $s_n \to s$ and $\alpha s = u(x)$ for some $x \in \hat{X}$ and $\alpha > 0$. Pick $0 < \beta < \alpha$ and suppose, without loss of generality, that $\beta s_{ni} < \alpha s_i$ for all n and any $i \in N$ with $s_i > 0$. For any i with $s_i = 0$ let $u_i(\delta_{ni}\omega) = \beta s_{ni}$. Put $\delta_n = \sum_{s_i=0} \delta_{ni}$. Then $\delta_n \to 0$. Define an allocation y_n by $y_{ni} = \delta_{ni}\omega$ if $s_i = 0$ and $y_{ni} = (1 - \delta_n)x_i$ otherwise. We have that, for sufficiently large n, $u_i(y_{ni}) \geqslant \beta s_{ni}$ for every i. So we have found $y \in \hat{X}$ such that $U(y) \geqslant \beta s_n$ componentwise. Let now $\mu_i \in [0, 1]$ be such that $u_i(\mu_i y_i) = \beta s_{ni}$. Then $\sum_{i \in N} \mu_i y_i \leqslant \sum_{i \in N} y_i \leqslant \omega$; i.e., we have a feasible allocation which has a utility image that is precisely βs_n. Since $\beta < \alpha$ is arbitrary we conclude that $\lim_{s_n \to s} \inf f(s_n) \geqslant f(s)$.

The proof that $\lim_{s_n \to s} \sup f(s_n) \leqslant f(s)$ is analogous and even simpler.

Q.E.D.

With no change in the proof, the above proposition remains valid if preferences are restricted to consumption sets which are closed, convex subsets of L_+ containing the origin and ω.

A feasible allocation $x \in X^N$ is *weakly efficient* if there is no other feasible allocation x' such that $x'_i \succ_i x_i$ for every $i \in N$. If x is weakly efficient then $U(x)$ must belong to the upper frontier of $U(\hat{X})$ and for

$$s = \frac{1}{\sum_{i \in N} u_i(x_i)} U(x)$$

the value $f(s)s$ is attained by $U(\hat{X})$. However, with the hypotheses so far there is no reason for the value $f(s)$ to be attained for every s, or equivalently for $U(\hat{X})$ to be closed. In other words, while weakly efficient allocations exist trivially (just give the entire ω to any consumer), interesting ones, e.g., with some prescribed sharing of "utility," are not guaranteed to exist.

EXAMPLE 2 (Araujo (1985)): Let $L = l_\infty$ be the space of bounded infinite sequences endowed with the sup norm. Take $\omega = (1, \dots, 1, \dots)$ and $N = 2$. The preferences of $i=1$ (resp. $i=2$) are given by $x \succsim_i y$ if and only if $\lim_{n\to\infty} \inf x_n \geqslant \lim_{n\to\infty} \inf y_n$ (resp. $\sum_{n=1}^{\infty} (1/2^n)x_n \geqslant \sum_{n=1}^{\infty} (1/2^n)y_n$). It is simply seen that the set $U(\hat{X})$ has then the form indicated in Figure 3 (the point (0, 1) is included). Note that the first consumer is absolutely patient while the second discounts the future.

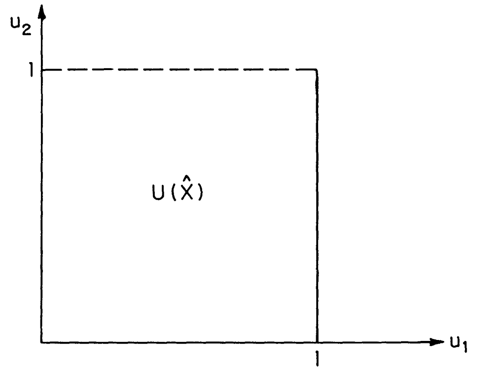

FIGURE 3

As emphasized by Araujo (1985) we can hardly expect to prove the existence of equilibrium prices in our (finite number of consumers) context if our model does not guarantee in any generality the existence of weakly efficient allocations. What is needed is that we be able to replace sup. by max. in the definition of $f: \Delta \to [0, N]$ or, equivalently in view of Proposition 2, that $U(\hat{X})$ be closed. This is what we shall assume (we give the condition directly in terms of preferences).

CLOSEDNESS HYPOTHESIS: Consider a sequence $x_n \in \hat{X}$ and suppose that $n > m$ implies $x_{ni} \succsim_i x_{mi}$ for all i. Then there is $x \in \hat{X}$ such that $x_i \succsim_i x_{ni}$ for all n and $i \in N$.

It is simply seen that the above hypothesis is equivalent to the closedness of $U(\hat{X})$. Of course, the Closedness Hypothesis would not be very good if one could not find conditions on individual preferences which yield it. But this is easy. Let the sequence x_n be as in the hypothesis. Then $V_n = \bigcap_{i\in N} \{x \in X^N: x_i \succsim_i x_{ni}\} \cap \hat{X}$ constitutes a nested sequence of closed sets. If $\bigcap_n V_n$ is nonempty then we are done. This intersection will be nonempty if in a topology for which the convex sets $\{z \in X: z \succsim_i y\}$, $i \in N$, $y \in X$, are closed, the set $\hat{X}$, which is convex, bounded below by 0 and above by ω, is compact. If L is dual to some other space B then $\hat{X}$ is compact in the weak-star topology (i.e., the topology of the pointwise convergence when members of L are looked at as linear functionals on B). For reflexive spaces such as L_p, $1 < p < \infty$, the weak-start coincides with the weak topology. Because sets which are norm closed and convex are weakly closed, the closedness hypothesis is automatically satisfied on them. For L_∞ and $ca(K)$, which have preduals but are not reflexive, the hypothesis that the sets $\{z \in X: z \succsim_i y\}$ are weak-star closed is strong but it has clear and natural economic interpretations. In L_∞ it means that preferences "discount" the future (see Bewley (1972), and Brown-Lewis (1982)) while in $ca(K)$ it says that commodity bundles with similar characteristics are treated similarly by preferences (see Mas-Colell (1975), and Jones (1984a)). The space L_1 has no predual but it is nonetheless still true that $\hat{X}$ is weakly compact (because it is order-bounded; I owe this observation to C. Aliprantis).

7. SUPPORTING PRICES FOR WEAKLY EFFICIENT ALLOCATIONS

This is the key section of this paper. Suppose that individual preferences are continuous, monotone, and convex and that x is a weakly efficient allocation. We will look for conditions guaranteeing the existence of a supporting price functional, i.e., a $p \in L_+^*$, $p \neq 0$, such that $z \succsim_i x_i$ implies $p \cdot z \geqslant p \cdot x_i$ for every i. In the next section we shall see that once these supporting prices are available, the rest of the existence proof is fairly routine sailing.

Obviously, we shall have to require that the preferences of individual consumers be proper. As we saw in Section 4, properness of $\succsim_i$ at x_i is equivalent to the existence of $p_i \in L_+^*$, $p_i \neq 0$, such that $z \succsim_i x_i$ implies $p \cdot z \geqslant p \cdot x_i$. One may have hoped that, with x a weak optimum, properness, or at least uniform properness, would imply social supportability (i.e., "social properness"). Unfortunately, this does not follow automatically as the following example, due to Jones (1984b), demonstrates. Experts in urban economics will have no difficulty in recognizing it.

EXAMPLE 3 (Jones): Let $L = L_\infty([0, 1])$ have the topology induced by the duality with $C^1([0, 1])$, i.e., $y_n \to y$ if and only if $\int_0^1 y_n(t)g(t)\,dt \to \int_0^1 y(t)g(t)\,dt$ for every continuously differentiable g. Let $N = 2$ and the preferences of $i = 1$ (resp. $i = 2$) be represented by $u_1(x_1) = \int_0^1 (1-t)x_1(t)\,dt$ (resp. $\int_0^1 tx_2(t)\,dt$). The utility functions are linear and continuous. Hence, preferences are uniformly proper. Put $\omega(t) = 1$ for all t. The feasible allocation x defined by $x_1(t) = 1$ if $t \leqslant \frac{1}{2}$, $x_1(t) = 0$ if $t \geqslant \frac{1}{2}$ (resp. $x_2(t) = 0$ if $t \leqslant \frac{1}{2}$, $x_2(t) = 1$ if $t \geqslant \frac{1}{2}$) is weakly optimal. However, it cannot be supported by any continuous linear functional since the only supporting functional, which is given by $q(t) = 1 - t$ if $t \leqslant \frac{1}{2}$, $q(t) = t$ if $t \geqslant \frac{1}{2}$, cannot be expressed as a continuously differentiable function of t and, therefore, it is not continuous as a linear functional on L.

The main result of this section is that if the commodity space has the structure of a *topological vector lattice* then individual uniform properness implies social supportability.

An ordered linear space is a *vector lattice* (or Riesz space) if the order $\geqslant$ of the space is a lattice order, i.e., for any $x, y \in L$ there are elements of the space, denoted $x \vee y$, $x \wedge y$, such that $z \geqslant x$, $z \geqslant y$ (resp. $z \leqslant x$, $z \leqslant y$) implies $z \geqslant x \vee y$ (resp. $z \leqslant x \wedge y$). For any $x \in L$ one can then define the positive part $x^+ = x \vee 0$, the negative part $x^- = (-x) \vee 0$, and the absolute value $|x| = x^+ + x^-$.

If, in addition, the order and topological structures of the space fit together well enough for the lattice operations to be uniformly continuous, then we have a *topological vector lattice* (or locally solid Riesz space). A salient and all important example is the *Banach lattices*, which are vector lattices admitting a complete norm $\| \ \|$ with the property that $|x| \leqslant |y|$ implies $\|x\| \leqslant \|y\|$. The commodity space of Example 3 fails to be a topological vector lattice.

Although positive cones inducing lattice orders are mathematically special (for example, if $L=R^n$ then L_+ is a lattice cone if and only if it is generated by precisely n extreme rays), economic examples (including Example 3 above) fall naturally into a vector lattice framework. We do not view, therefore, this part of the hypothesis as restrictive. It is quite another matter with the requirement that the lattice operations be continuous. It is here that the strength of the hypothesis lies. We shall defer a more detailed discussion of this point until after the next Proposition.

PROPOSITION 3: *Suppose that L is a topological vector lattice and that the preference relations $\succsim_i$, $i \in N$, on L_+ are convex, monotone and uniformly proper. For each $i \in N$ let $v_i \in L_+$ be as in the definition of properness. Put $v = v_1 + \cdots + v_N$. Then there is $V \subset L$, an open neighborhood of zero, such that any weakly optimal allocation can be supported by a $p \in L_+^*$ with $p \cdot v = 1$ and $|p \cdot z| \leqslant 1$ for all $z \in V$.*

PROOF: For every i let $V_i \subset L$ be the neighborhood of zero given in the definition of uniform properness. Without loss of generality we can assume that V_i is convex (the space L is locally convex) and that $V_i = -V_i$. Put $V = V_1 \cap \cdots \cap V_N$ and let $\Gamma \subset L$ be the open, convex cone spanned by $\{v\} + V$.

For a given weakly optimal allocation x define

$$Z = \left\{ \sum_{i=1}^{N} (z_i - x_i) \colon z \succsim_i x_i \right\} \subset L.$$

From this point on, the proof proceeds in two steps. The first shall establish the collective properness property, namely, that for an adequately chosen Γ we have $Z \cap (-\Gamma) = \varnothing$. The proof does not require the convexity of preferences. It relies crucially on the decomposition property for vector lattices and on the (uniform) continuity of the lattice operations. So the full strength of the topological vector lattice assumption is used in the first step. The second step is then a straightforward separation argument relying on the Hahn–Banach theorem; convexity of preferences is, of course, essential here.

STEP 1: Because the lattice operations are uniformly continuous, we can choose the sets V_i so that they are *solid*, i.e., $|u| \leqslant |z|$, $z \in V_i$, implies $u \in V_i$ (see Aliprantis and Burkinshaw (1978, p. 34)). To show that $Z \cap -\Gamma = \varnothing$ we argue by contradiction. Suppose there are $z_i \succsim_i x_i$ such that, denoting $\omega = \sum_{i=1}^N x_i$ and $z = \sum_{i=1}^N z_i$, we have $z - \omega \in -\Gamma$, i.e., $z - (\omega - \alpha v) \in \alpha V$ for some $\alpha > 0$. Call $y = \omega - \alpha v$. We claim that $(y-z)^- \leqslant z + \alpha v$. This can be seen as follows. Because $z \geqslant 0$ we have $y - z \leqslant y \leqslant \omega$. Hence, $(y-z)^+ = (y-z) \vee 0 \leqslant \omega$. So,

$$z = y - \omega + \omega - (y-z)^+ + (y-z)^- \geqslant -\alpha v + (y-z)^-.$$

Because $z + \alpha v = \sum_{i \in N} (z_i + \alpha v_i)$ the Decomposition Property of vector lattices (see, for example, Aliprantis and Burkinshaw (1978, p. 3)) implies that we can write $(y-z)^- = \sum_{i \in N} s_i$ where $0 \leqslant s_i \leqslant z_i + \alpha v_i$ for each $i \in N$. Define now $z_i' = z_i + \alpha v_i - x_i \geqslant 0$. Suppose that, for some i, $z_i \succsim_i z_i'$. Because of the properness property this implies $s_i \notin \alpha V$. On the other hand, $0 \leqslant s_i \leqslant (y-z)^- \leqslant |y-z|$ and

$y-z \in \alpha V$. Hence, $s_i \in \alpha V$ and we have a contradiction. Therefore, $z_i' \succ_i z_i \succsim_i x_i$ for every i. But this contradicts the weak efficiency of x because

$$\sum_{i=1}^{N} z_i' = z + \alpha v - (y-z)^- \leqslant z + \alpha v - (y-z)^- + (y-z)^+$$
$$= z + \alpha v + (y-z) = y + \alpha v = \omega = \sum_{i=1}^{N} x_i.$$

STEP 2: Because of the convexity hypothesis on preferences, the open set $Z + \Gamma$ is convex. Also, $0 \notin Z + \Gamma$ because $Z \cap -\Gamma = \varnothing$. Therefore, by the Hahn–Banach Theorem (see Schaefer, 1971, p. 46) there is a nonzero, continuous linear functional p such that $p \cdot y > 0$ for all $y \in Z + \Gamma$. Remembering that Γ is a cone and $0 \in Z$ this yields $p \cdot z \geqslant 0$ for all $z \in Z$ and $p \cdot y > 0$ for all $y \in \Gamma$. In particular, $p \cdot v > 0$. So, without loss of generality, we can take $p \cdot v = 1$. If $z \in V$ then $v - z \in \Gamma$ and so $p \cdot z \leqslant p \cdot v = 1$. Since the same applies to $-z$ we get $|p \cdot z| \leqslant 1$. Finally, let $z \succsim_i x_i$. Then $z - x_i \in Z$ and so, $p \cdot z \geqslant p \cdot x_i$. We conclude that p supports x. The fact that $p \geqslant 0$ follows from the monotonicity of any $\succsim_i$. *Q.E.D.*

The requirement that the lattice operations be uniformly continuous is not minor. It places strong restrictions on the topology of L. For example, if L is a Banach lattice then the lattice operations are uniformly continuous in the weak topology if and only if the space is finite dimensional (Aliprantis and Burkinshaw (1978, p. 42)). Of course, if L is a Banach lattice then Proposition 3 gives us in all generality a price functional in the norm dual. If L is reflexive then this is precisely what one wants. But if as L_∞ and $ca(K)$ the space L has a predual $\hat{L}$ strictly smaller than its dual L^* then we may ideally want a price functional in $\hat{L}$, i.e., in L_1 for L_∞ and in $C(K)$ for $ca(K)$. One way to aim at this is to work with a topology on L generating $\hat{L}$ as dual. It is by no means guaranteed that such a topology, weaker than the norm, will preserve the continuity of the lattice operations. Consider, for example, the sequence $x_n = \delta_{(1/n)} - \delta_0$ in $ca([0,1])$; it converges to 0 in the weak-star topology but $x_n^+ \to \delta_0 \neq 0$. The space L_∞ is again better behaved: the Mackey topology induced by its pre-dual does the trick (I owe this observation to C. Aliprantis and W. Zame). In spite of all this, Proposition 3 is still quite relevant because by using a price system in the norm dual it is often possible to strengthen the properties of the price functional through space-specific arguments.

We end this section with four remarks:

(i) One feels there is still considerable room for improvement between the level of pathology of Example 3 and the strength of the continuity hypothesis of Proposition 3. See Mas-Colell (1986) for an exploration of this gap.

(ii) Even to establish supportability at a single weak optimum, the proof of Proposition 3 makes essential use of uniform properness as opposed to properness at a point.

(iii) Over and above the commodity space being a topological vector lattice, it is also important for Proposition 3 that the consumption set be the positive orthant (or at least an order-closed subcone of the positive orthant containing v_i). Relaxing this would be of interest, mostly for the insights it would provide into the generalization to the production case.

(iv) It is to be noted that the lattice theoretic hypothesis on the space and the consumption set cannot be dispensed with even in the finite-dimensional case. While the existence of a collective supporting price p follows purely from convexity considerations, this is not so for the requirement $p\cdot v=1$. For an investigation of the finite-dimensional case see Yannelis–Zame (1984, Sec. 7) and Mas-Colell (1985a).

8. AN EQUILIBRIUM EXISTENCE THEOREM

All the ingredients are not in place for the existence proof.

Suppose that $\omega=\sum_{i\in N}\omega_i$, $\omega_i\geqslant 0$, i.e., every consumer has a specific claim on a part of the initial endowment vector. A feasible allocation $x\in L_+^N$ and a $p\in L_+^*$, $p\neq 0$, constitute a *quasiequilibrium* if, for all i, $p\cdot\omega_i=p\cdot x_i$ and $p\cdot z\geqslant p\cdot x_i$ whenever $z\succsim_i x_i$. An *equilibrium* if in fact $z\succsim_i x_i$ implies $p\cdot z>p\cdot x_i$ for all i. The latter property holds at a quasiequilibrium for any i such that $p\cdot\omega_i>0$.

THEOREM: *Let L be a topological vector lattice and $\succsim_i$, $i\in N$, a collection of monotone, convex, continuous and uniformly proper preference relations on the positive orthant of L. Suppose that for the economy $\{(\succsim_i,\omega_i)\}_{i\in N}$ the vector $\omega=\sum_{i=1}^N\omega_i$ is desirable for every $i\in N$ and that the Closedness Condition holds. Then there is a quasiequilibrium (x,p). Moreover, $p\cdot v=1$ where $v=v_1+\cdots+v_N$ and v_i is the properness vector for $\succsim_i$.*

PROOF: The demonstration is a straightforward adaptation (in fact, almost a transcription) of Bewley's (1969) and Magill's (1981) proof for the case L_∞. It consists of three steps. In the first we define a certain correspondence $\Phi:\Delta\to T=\{z\in R^N:\sum_{i\in N}z_i=0\}$ which has the property that any of its zeroes yields a quasiequilibrium. In the second we show that Φ is convex valued and upper-hemicontinuous. Finally in the third we prove that Φ has a zero.

STEP 1: Take $f:\Delta\to[0,N]$ as in Section 6 and $v\in L_+$, $V\subset L$, as in Proposition 3. For any $s\in\Delta$ pick a feasible allocation $x(s)$ such that $U(x(s))=f(s)s$. We can, without loss of generality, assume that $\sum_{i\in N}x_i(s)=\omega$ (indeed, not everybody can be made better off, which means that if there is any surplus left it can be distributed so as to make no one better off). Then let $P(s)=\{p\in L_+^*: p\cdot v=1, |p\cdot z|\leqslant 1$ for all $z\in V$, p supports the weakly efficient allocation $x(s)\}$. Obviously, $P(s)$ is convex and, by Proposition 3, nonempty. Finally, put $\Phi(s)=\{(p\cdot(\omega_i-x_i(s)),\ldots,p\cdot(\omega_N-x_N(s))): p\in P(s)\}$. Then $\Phi(s)$ is non-empty and convex valued, $\sum_{i\in N}z_i=0$ for any $z\in\Phi(s)$, and $0\in\Phi(s)$ if and only if $x(s)$ is a quasiequilibrium.

STEP 2: This is the key step. We establish that $\Phi:\Delta\to T$ is an upper hemicontinuous correspondence. Let $s_n\to s$, $w_n\in\Phi(s_n)$, $w_n\to w\in T$. We should show that

$w \in \Phi(s)$. Suppose that $p_n \in P(s_n)$ is the price vector yielding w_n. Because $|p_n \cdot z| \leq k$ for all $z \in V$ we can assume, by Alaouglou's Theorem (see Schaefer (1971, p. 84)), that p_n has a weak-star limit, i.e., $p_n \cdot y \to p \cdot y$ for any $y \in L$. This yields $p \cdot v = 1$. Also, let $z \succsim_i x_i(s)$. Because $f(s_n)s_n \to f(s)s$ we should eventually have $z \succsim_i x_i(s_n)$ which implies $p_n \cdot z \geq p_n \cdot x_i(s_n) = p_n \cdot \omega_i - w_{ni}$. Taking limits, $p \cdot z \geq p \cdot \omega_i - w_i$. Because of monotonicity we can conclude that $p \cdot z \geq p \cdot \omega_i - w_i$ whenever $z \succsim_i x_i(s)$. In particular, $p \cdot x_i(s) \geq p \cdot \omega_i - w_i$ for all i. Since $\sum_{i \in N} x_i(s) = \sum_{i \in N} \omega_i$ we in fact have $p \cdot x_i(s) = p \cdot \omega_i - w_i$. This proves that $w \in \Phi(s)$.

STEP 3: The proof that $\Phi : \Delta \to T$ has a zero is routine. We simply consider the upper hemicontinuous, nonempty, convex valued map $s \mapsto s + \Phi(s)$ on Δ and note that it satisfies suitable boundary conditions for an application of Kakutani's fixed point theorem. Indeed, $s_i = 0$ and $w \in \Phi(s)$ implies $w_i \geq 0$ because $u_i(x_i(s)) = 0$, i.e., $0 \succsim_i x_i(s)$, and so $0 \geq p \cdot x_i(s) \geq 0$ for any p supporting $x(s)$. *Q.E.D.*

The Theorem would not be of much interest if we could have $p \cdot \omega = 0$. The way to avoid this sort of degeneracy is to make the natural assumption that, for all $i \in N$, ω is desirable in the (relatively) strong sense of being able to take $v = \omega$. Then the Theorem guarantees $p \cdot \omega = 1$. This, incidentally, is not an issue specific to the infinitely many commodities context. It can arise with one consumer and two commodities.

Finally, a comment on the initial motivation for this research, the space $ca(K)$. The Theorem makes clear that the bulk of smoothness hypotheses of Jones (1984a) is needed to get the price system in $C(K)$. Only a minor part of them (roughly, the part that does not depend on the topological structure of K) is required for the preferences to be proper with respect to the norm topology and to get, therefore, a price system in the norm dual.

9. EXTENSIONS AND RELATED RECENT WORK

Yannelis and Zame (1984) have reproved and extended (to unordered, locally nonsatiated preferences) the Theorem of Section 8 using a technique of approximation by finite-dimensional economies. In a more limited context, a different proof of the same general nature is outlined in Brown (1983). An enlightening elucidation of the role of the uniform properness condition is contained in Aliprantis-Brown-Burkinshaw (1985).

Our Theorem depends on the consumption sets being the positive orthant of the space. This suggests that the extension to production economies is a nontrivial task. Duffie-Huang (1983b), Duffie (1984), and Jones (1984c) have provided generalizations to production. All assume some form of noninteriority of the production set. With this (strong) hypothesis the theory is quite parallel to the L_∞ case. Properness assumptions are dispensable and weak optima are supportable (this was proved by Debreu (1954a)). The existence theorems of Aliprantis-Brown (1983), Yannelis (1984), and Simmons (1985), where excess demand functions are primitives, could also be interpreted in terms of noninteriority assumptions on production sets. A model where the consumption rather than the production sets have nonempty interior, is in Chichilnisky-Heal (1984); see also Kreps (1981). Recently, Zame (1985) has succeeded in finding an ingenious way to exploit lattice theoretic properties in order to obtain a Theorem which does incorporate nontrivial production while avoiding a nonempty interior hypothesis. See also Mas-Colell (1986).

Another topic for further research, the weakening of the continuity hypothesis on the lattice operations, has already been mentioned in Section 7.

Our method of proof is so dependent on the finiteness of the number of agents that it prompts one to ask how the double infinity could be handled in an abstract setting. An important reference is Ostroy (1984) where a result for the commodity space L_1 is obtained. The problem tackled in Section 7 does not arise in Ostroy's work because, in effect, the properness-like hypothesis is imposed directly at the collective level. This is, of course, well in accord with the dictum that it is best to face problems one at a time.

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