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Nash equilibrium and economics: Remarks

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The concept of Nash equilibrium has become central to the theory and practice of economics. In these remarks I would like to reflect a bit on why this has come to happen.

Interestingly enough the founding treatise of the theory of games (*The Theory of Games and Economic Behaviour*, published in 1944 and authored by a mathematician J. von Neumann and an economist O. Morgenstern) had, for practically a quarter of a century, very little influence in economics. And not because it was not known. Leo Hurwicz (who has just been awarded the 2007 Economics Nobel Prize for the launching of mechanism design theory, to which we will refer later on) wrote a very good and extensive review for it in the *American Economic Review*, the leading economics journal of the day. There is a conventional explanation on why this occurred and since I do not have any better insight to offer here it goes: Von Neumann and Morgenstern focused their efforts on the pure conflict case, that is, on zero-sum games. For two player zero-sum they developed a very sharp theory (the minmax theorem) that answered the question: if a game is played once, what should a player do. The problem from the point of view of economics is that zero-sum is by no means the leading case. Economics from its inception has been about how to realize gains from trade and how to articulate mutually beneficial agreements. What one gains may be somebody else loss but it does not need to. Very often everybody can gain, and economics is about that. Certainly, there may be conflict on how to split gains from trade but what makes economics interesting is the interplay between this and the realization of the gains from trade.

There is a related difficulty, which, in fact, already arises in the context of J. von Neumann. Zero-sum theory is restricted to two players. As soon as there are more players the possibility for simultaneous gains for two of them arises and therefore the situation includes incentives to collude, i.e. to coordinate. From this observation J. von Neumann built a general theory on “cooperative” lines. Cooperative theory, especially through the concept of the core, introduced in 1953 by L. Shapley and D. Gillies, was extensively applied to some areas of economics, starting with M. Shubik in

1955. Interestingly the concept had been anticipated within economics by F.Y. Edgeworth in 1881!. It is puzzling that J. von Neumann did not formulate the core concept since he was responsible for the notion of imputation domination on which it is, straightforwardly, based. It is possible that he wanted much more – the core, for example, may be empty – and thus proposed and focused instead on the more sophisticated notion of “stable solution”, that lead essentially nowhere.

But it is even more curious and surprising that in J. von Neumann treatise there is no inkling whatsoever of the notion of non-cooperative equilibrium and of the solution that would be introduced by Nash in his thesis (and published in the *Proceedings of the Nat. Acad. of Science* in 1950). Specially if one takes into account that they developed their work with economic applications in mind and that, as in the Core, the notion of Nash equilibrium had again been anticipated within economics, by Cournot in 1838. At any rate, it was the concept of Nash equilibrium that, first slowly and then as a torrent, come to become the underpinning of economic theory and, therefore, the entry corridor for game theory into economics, both in its positive and normative facets.

I believe that the reason for the success of Nash equilibrium is that it fits extraordinarily well the intellectual outlook and the traditions of economics. Let me explain, concentrating first on the positive (predictive) facet of the discipline. At the end I shall make a comment on the normative.

Nash equilibrium can be viewed as the union of two classic notions of economics: maximization and perfect foresight. Indeed, a state of an economy or of a game is in (Nash) equilibrium if, first, every agent takes a utility maximizing action given his expectations of what the others will do, and second, everyone is correct in the expectations of what others will do, or equivalently every one ends up doing what is expected of him/her. Somehow expectations are coordinated.

The second notion is harder to take than the first. Thus, another great mathematician, Poincaré, in correspondence with the founder of the theory of general economic equilibrium, Leon Walrás, pointed out that, in his view, economics assumed that individuals were “infiniment égoïstes” (this is not exactly the same that utility maximization but, as a first approximation, we could assume that it is) and “infiniment clairvoyantes” (correct expectations could be viewed in this light). He added that he could accept, as a simplification the first, but that he found difficult to accept the second.

In spite of this fact, economist come to propose and to accept this coordinated foresight in good because of two considerations. In the first place, they were not thinking of one-shot affairs but of repetitive situations, where their equilibria could be thought as the stationary states of simple, implicit, dynamics. Therefore a candidate for the coordination mechanism that correct expectations seems to require was at hand: it was history. In the second place, classical economics contemplated mostly the polar opposite to the case of a two player games, namely a world with multitudes of small traders. Then it makes sense to view the individual agent as devoid of the need to be strategic: whatever s/he does is no significant import to the rest of the world and so s/he can as well assume that the latter will not react to her deviations. That is, agents expect that the environment will remain unchanged tomorrow and therefore adapt optimally to it. Equilibrium is then the state in which this adaptation means that all players stay put, i.e. a stationary state.

But the theory of games does not admit muddles. A one-shot game is one thing and a multi-shot one is another. The standard set by J. von Neumann in this respect was very clear. The solution attained in two person zero-sum games, the minmax, was a one shot affair and, although the minmax strategies are in Nash equilibria, this was not its most relevant characteristic. It was more important to observe that in order to play them, no precise expectations are needed on how the opponent will play: it is enough to be prudent and protect oneself against the worse (and it does not take much thinking to realize that it pays to be prudent). It is likely that in contemplating the several players case J. von Neumann maintained the same desiderata and that, in line with Poincaré, were not in the frame of mind that would have lead them to think naturally on the notion of Nash equilibrium. Because it is indeed a very daring intellectual jump to postulate that players playing once and independently will do so under compatible expectations that besides will be correct once maximization took place under them. If one thinks about it, and thinks deeply (and modern game theory has thought very deeply about that!) one realizes that the concept is immensely subtle and the jump awesome: it was the great merit of Nash having made it. The concept is formally simple but it took intellectual courage to propose it within the context of the game theory of its day. Besides he found himself with an added bonus: a general existence theorem. Nash equilibria (possibly in mixed strategies) always exist. It is interesting that this followed from the application of fixed-point results that had previously been used by von Neumann in some work in growth theory and in the first proof of the minmax theorem. The founders

had therefore all the ingredients to have come up with the notion of equilibrium. But they did not. It was left to Nash. An interesting question is if Nash was familiar with the concept of economic equilibrium (described informally earlier). Nash was, at that time, a mathematician. He had followed at least an undergraduate course in economics. May be this colored his vision in a fertile direction!

As explained a moment ago, the notion of (Nash) equilibrium is easier to accept if every individual player is insignificant (this, of course, can be defined precisely). This is a common feature in economics (mass markets) but one can imagine that it would have been considered an uninteresting case by J. von Neumann, at least for the purposes of non-cooperative analysis. If one approaches the theory of games from the vantage point that gives to it its name (parlor games) it is evident that the strategic interplay among players, all of them significant, is the essence.

In summary we could therefore explain the success of Nash equilibrium in economic as follows. Nash equilibrium abstracts and formalizes some key features of an informal notion of economic equilibrium thought for a repeated world with individual insignificant players. This abstraction has two payoffs: as a concept it applies to any game, and it exists in any game. Once the jump is made, a world of possibilities open up for the economic analyst, because economic reality is not only mass markets, it is also big firms, governments, trade unions, that is, economic life depends also on the actions of non-insignificant players. In spite of the brilliant precedent of Cournot, economics had been stumbling. It lacked a systematic and consistent methodology to analyze such situations. Nash equilibrium provided it. It is, to repeat, a subtle concept and it was not easy to get used to. It took some time to catch fire (around 20 years) but eventually it did.

A final remark on the normative theory of economics, where again the concept of Nash equilibrium is at center stage. The reason is simple. Suppose that an Authority has designed a mechanism of social interaction (formally, a game) with the property that whatever the characteristics of the population the “outcome” of the mechanism is good for that population according to some desired criterion. But what is the “outcome”? It does not take much reflection to convince oneself that it must be a Nash equilibrium of the game, or, using a term coined by L. Hurwicz, “incentive compatible”.

Together with L. Hurwicz, the 2007 Economics Nobel was awarded to E. Maskin and R. Myerson for, as we already mentioned, mechanism design. So it is fitting to refer to the splendid textbook of the latter [1] for all the issues treated in these remarks.

REFERENCES

- [1] R. Myerson, *Game Theory: Analysis of Conflict*, Harvard University Press, 1991.

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